



Total No. of Questions : 5

Total No. of Printed Pages : 2

Roll No.

JD-309

M.Sc. , I Semester Examination,
December-January : 2025-2026

MATHEMATICS

(Advanced Abstract Algebra - I)

Paper - I

Time : Three Hours]

[Maximum Marks : 80

Note : Answer any two parts from each question. All questions carry equal marks.

1.
 - a) Show that every nilpotent group is solvable, but the converse is not true.
 - b) Let G be a nilpotent group. Then every subgroup of G and every homomorphic image of G are nilpotent. Prove it.
 - c) Show that the symmetric group S_3 of degree 3 is solvable.

2.
 - a) Let K is algebraic extension of F and L is algebraic extension of K . Then show that L is algebraic extension of F .
 - b) Let a and b are the two separable elements in some extension of a field F . Prove that \exists an element G in the same extension such that $F(C) = F(a, b)$.
 - c) Let E is finite extension of F , then E is a normal extension of $F \Leftrightarrow$ it is splitting field of a polynomial $f(x) \in F(x)$.

3.
 - a) Let G' be the field of complex numbers and R the field of real numbers. Show that G' is a normal extension of R .
 - b) If $f(x) \in F(x)$ is an irreducible polynomial over a finite field F , then show that all the roots of $f(x)$ are distinct.
 - c) Let E be a finite extension of F , then E is a normal extension of F if and only if E is a splitting field of some polynomial over F .



4. a) If F is a finite extension of a field E , then prove that $G(E|F)$ is a finite group and $|G(E|F)| \leq [E : F]$.
- b) Prove that, the Galois group of $x^4 - 2 \in \mathbb{Q}[x]$ is the octic group.
- c) State and prove fundamental theorem of Galois theory.
5. a) Prove that $f(x) \in F(x)$ is solvable by radicals over F if and only if its splitting field E over F has solvable Galois group $G(E|F)$.
- b) Show that the polynomial $2x^5 - 5x^4 + 5$ is not solvable by radicals.
- c) Show that if an irreducible polynomial $p(x) \in F(x)$ over a field F has a root in a radical extension of F , then $p(x)$ is solvable by radicals over F .

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M.Sc., I Semester Examination,
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MATHEMATICS

(Real Analysis - I)

Paper - II

Time : Three Hours]

[Maximum Marks : 80

[Minimum Pass Marks : 16

Note : Answer any two parts from each question. All questions carry equal marks.

Unit-I

1. a) State and prove M_n test for uniform convergence of sequence of function.
b) Show that the sequence $\{f_n\}_{n=1}^{\infty}$ where,

$$f_n(x) = \frac{nx}{1+n^2x^2} \text{ does not converges uniformly on } \mathbb{R}.$$

- c) Let $\{f_n\}$ be a sequence of real valued functions on a metric space (X, d) which converges uniformly to the function f on X . If each f_n ($n = 1, 2, 3, \dots$) is continuous on X then prove that f is also continuous on X .

Unit-II

2. a) Obtain the radius of convergence R of the power series $\sum c_n z^n$, where

i) $c_n = \frac{2^n}{n^2}$

ii) $c_n = \frac{n^3}{3^n}$

- b) State and prove Abel's theorem (first form).
c) State and prove Tauber's theorem.



Unit-III

3. a) State and prove Inverse function theorem.
 b) Prove that a linear operator Λ on a finite dimensional vector space X is one to one iff and only if the range of Λ is all of X , that is iff Λ is onto.
 c) Let f maps a convex open set $E \subset \mathbb{R}^n$ into \mathbb{R}^m , f be differentiable in E and there be a real number M such that $\|f'(x)\| \leq M$ for every $x \in E$ then prove that $|f(b) - f(a)| \leq M|b - a|$ for all $a \in E, b \in E$.

Unit-IV

4. a) Find the shortest distance from the point $\left(\frac{3}{2}, 0\right)$ to the parabola $y^2 = 4x$.
 b) Let E be an open subset of \mathbb{R}^n and $f: E \rightarrow \mathbb{R}$ is a ϵ' mapping. Let $g: E \rightarrow \mathbb{R}^m$ be a ϵ' mapping such that $g'(x)$ is onto for every x in $Y = g^{-1}(0)$. Assume that $n > m$. If f has an extremum on Y at γ , then prove that there is a linear function $\lambda: \mathbb{R}^m \rightarrow \mathbb{R}$ such that γ is a stationary point for $f - \lambda \circ g$.
 c) If $x + y + z = u, y + z = uv, z = uvw$, then prove that

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2v$$

Unit-V

5. a) Let E be an open set in \mathbb{R}^n , T be a ϵ' mapping of E into an open set $V \subset \mathbb{R}^m$. Let w and λ be k form and m form in V , respectively. Then prove that

$$(w + \lambda)_T = w_T + \lambda_T \text{ if } k = m$$

 b) Suppose T is a ϵ' mapping of an open set $E \subset \mathbb{R}^n$ into an open set $V \subset \mathbb{R}^m$, s is a ϵ' mapping of V into an open set $W \subset \mathbb{R}^p$ and w is a k -form in w , so that w_s is a k -form in V and both $(w_s)_T$ and w_{ST} are k -form in E , where ST is defined by $(ST)(x) = S(T(x))$. Then prove that

$$(w_s)_T = w_{ST}$$

 c) Suppose w is a k -form in an open set $E \subset \mathbb{R}^n$, ϕ is a k -surface in E , with parameter domain $D \subset \mathbb{R}^k$ and Δ is the k -surface in \mathbb{R}^k , with parameter domain D , defined by $\Delta(u) = u(u \in D)$. Then prove that

$$\int_{\phi} w = \int_{\Delta} w_{\phi}$$

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MATHEMATICS

(Topology - I)

Paper - III

Time : Three Hours]

[Maximum Marks : 80

Note : Answer any **two** parts from each question. All questions carry **equal marks**.

1. a) Define well-ordered set. Prove that each set can be well ordered.
b) Define accumulation points and derived set. Let A be subset of a topological space, then prove that
$$\bar{A} = A \cup D(A)$$

c) Explain topological spaces with an example. Prove that intersection of two topologies is also topology. But union of two topologies is not necessarily a topology.
2. a) Give an example of first countable set. Prove that every second countable space is first countable and converse need not be true.
b) Define continuous mapping with an example. Let X and Y be topological spaces. Prove that a mapping $f: X \rightarrow Y$ is continuous if and only if the inverse image under f of every open set Y is open in X .
c) Define topology in terms of Kuratowski closure operator prove that a mapping f from a space X into another space Y is continuous if and only if

$$f(\bar{A}) \subset \overline{f(A)} \text{ for every } A \subset X$$



3. a) Define Hausdorff space with an example. Show that the space (\mathbb{R}, U) is T_3 .
- b) State and prove Tietze extension theorem.
- c) Define T_4 space with an example. Prove that every subspace of T_2 space is T_2 space.
4. a) Give an example of compact set. Prove that a subset A of \mathbb{R} is compact if and only if A is bounded and closed.
- b) Define sequentially and countably compact sets. Prove that a continuous image of a compact set (space) is compact.
- c) Explain compactification. Let (X^*, τ^*) be a one point compactification of a non-compact topological space (X, τ) . Then prove that (X^*, τ^*) is Hausdorff if and only if (X, τ) is Hausdorff and locally compact.
5. a) Give an example of disconnected sets. Prove that a continuous image of a sequentially compact set is sequentially compact.
- b) Define components. Let E be a connected subset of X . If F is a subset of X such that $E \subset F \subset \bar{E}$, then prove that F is connected. In particular, \bar{E} is connected.
- c) Explain connectedness on the real line. Prove that the component of a totally disconnected set X are singleton sets in X .

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MATHEMATICS

(Advanced Complex Analysis - I)

Paper - IV

Time : Three Hours]

[Maximum Marks : 80

[Minimum Pass Marks : 16

Note : Answer any two parts from each question. All questions carry equal marks.

1. a) Let $f(z)$ be analytic within and on the boundary C of a simply connected region D and let a be any point within C . Then show that

$$f'(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z-a)^2} dz$$

- b) State and prove Liouville's theorem.
c) Expand $\log(1+z)$ in a Taylor's series about $z=0$ and determine the region of convergence for the resulting series.

2. a) State and prove maximum modulus theorem.
b) State and prove Schwarz lemma.
c) Show that the equation $\dot{x}^4 + 4(1+i)z + 1 = 0$ has one root in each quadrant.



3. a) Define the residue at a singularity. Find the residue of $\frac{z^2}{z^2 + a^2}$ at $z = a_i$. State and prove Cauchy's residue theorem.
- b) Show that $\int_0^{2\pi} \frac{d\theta}{a + b \cos \theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$, $a > b > 0$
- c) Use the method of contour integration to prove that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$
4. a) Find all the Mobius transformations which transforms the unit circle $|z| \leq 1$ onto the unit circular disc $|w| \leq 1$.
- b) Show that the transformation $w = \frac{iz + 2}{4z + i}$ maps the real axis in the z -plane into a circle in the w -plane. Find the centre and the radius of the circle and the point in the z -plane which is mapped on the centre of the circle.
- c) Show that the transformation $w = \tan^2\left(\frac{\pi}{4}\sqrt{z}\right)$ transforms the interior of the unit circle $|w| = 1$ into the interior of a parabola.
5. a) State and prove Hurwitz's theorem.
- b) Show that a family F of holomorphic functions defined in a domain D that is $FC H(D)$ is normal if and only if F is locally bounded.
- c) State and prove Riemann mapping theorem.

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MATHEMATICS

(Advanced Discrete Mathematics - I)

Paper - V

Time : Three Hours]

[Maximum Marks : 80

[Manimum Pass Marks : 16

Note : Answer **any two** parts from each question. All questions carry equal marks.

Unit-I

1. a) Show that $p \leftrightarrow q \equiv (p \vee q) \Rightarrow (p \wedge q)$ using
 - i) Truth table
 - ii) Algebra of propositions .
- b) Convert natural language statements into predicate logic using quantifiers. Explain through an example.
- c) Prove that a semigroup $(S, *)$ is a group if and only if
 - i) there exist $e \in S$ such that $e * a = a$ for all $a \in S$
 - ii) for all $a \in S$ there exist $b \in S$ such that $b * a = e$.

Unit-II

2. a) Explain semigroups with an example.
- b) State and prove basic Homomorphism theorem.



- c) Write short notes about
- Direct Product
 - Congruence relation

Unit-III

3. a) Show that the set of natural numbers under divisibility forms a poset.
- b) For any a, b, c, d in a lattice (L, \leq) , if $a \leq b$ and $c \leq d$ then prove that
- $a \vee c \leq b \vee d$
 - $a \wedge c \leq b \wedge d$
- c) Let a, b, c be elements in lattice (L, \leq) . Then prove that
- $$a \vee (b \wedge c) \leq b \wedge (a \vee c)$$
- Where $a \leq b$.

Unit-IV

4. a) Find Karnaugh map and Simplify the expression $AB' + A'B'$
- b) Describe AND gate and OR gate with an example.
- c) Write short notes about the following :
- Atoms
 - Minterms

Unit-V

5. a) If $G = (\{0, 1\}, \{S\}, S, \{S \rightarrow 0S1, S \rightarrow 1\})$ is a grammar. Find $L(G)$.
- b) State and prove Kleene's Theorem.
- c) Write short notes about the following :
- Context free
 - Pumping Lemma

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